solutions

Question 1 (6 marks)

The table below shows the probability distribution for a random variable .

It is known that and .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |

(a) Determine the values of the constants and . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation using sum of probabilities  ✓ equation using expected value  ✓ determines  ✓ determines |

(b) Determine

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states value |

(ii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states value |

Question 2 (7 marks)

A calculator program will generate a single random integer , where . The program is run once, and the random variable is the number of fours or fives obtained.

(a) Explain why is a Bernoulli random variable. (1 mark)

|  |
| --- |
| **Solution** |
| In a single trial, will be or - either a or is generated () or not (). |
| **Specific behaviours** |
| ✓ explains event will or will not happen |

(b) Determine . (1 mark)

|  |
| --- |
| **Solution** |
| Require a or a : |
| **Specific behaviours** |
| ✓ correct probability |

(c) Determine the mean and standard deviation of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ mean   standard deviation |

The random variable is the number of fours or fives obtained in three consecutive runs of the program.

(d) Determine . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓     correct probability |

Question 3 (7 marks)

A farmer keeps a brood of hens that can each lay up to one egg per day. On any given day, the probability that a hen lays an egg is independent with a constant value of .

The discrete random variable is the number of eggs laid by the brood in one day and has a mean of 3 and standard deviation of .

(a) State the name given to this type of probability distribution and briefly explain why it is discrete. (2 marks)

|  |
| --- |
| **Solution** |
| Binomial. Discrete as can only be one of a specified set of values. |
| **Specific behaviours** |
| ✓ name   explanation |

(b) Determine the value of and the value of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes simultaneous equations   value of   value of |

(c) Determine the mean and variance of the distribution , where . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of   value of |

Question 4 (7 marks)

(a) Four random variables , , and are defined below. State, with reasons, whether the distribution of the random variable is Bernoulli, binomial, uniform or none of these.

(4 marks)

*The dice referred to is a cube with faces numbered with the integers 1, 2, 3, 4, 5 and 6.*

(i) is the number of throws of a dice until a six is scored.

|  |
| --- |
| **Solution** |
| Neither - distribution is geometric |
| **Specific behaviours** |
| ✓ answer with reason |

(ii) is the score when a dice is thrown.

|  |
| --- |
| **Solution** |
| Uniform - all outcomes are equally likely |
| **Specific behaviours** |
| ✓ answer with reason |

(iii) is the number of odd numbers showing when a dice is thrown.

|  |
| --- |
| **Solution** |
| Bernoulli - two complementary outcomes |
| **Specific behaviours** |
| ✓ answer with reason |

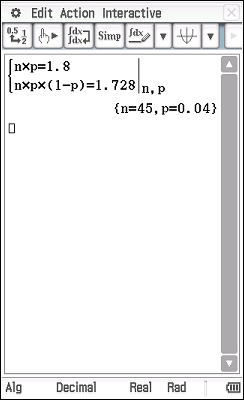
(iv) is the total of the scores when two dice are thrown.

|  |
| --- |
| **Solution** |
| Neither - distribution is triangular |
| **Specific behaviours** |
| ✓ answer with reason |

(b) Pegs produced by a manufacturer are known to be defective with probability , independently of each other. The pegs are sold in bags of for $4.95. The random variable is the number of faulty pegs in a bag.

If and , determine and . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes equations for mean and variance  ✓ solves for  ✓ solves for |



Question 5 (10 marks)

A slot machine is programmed to operate at random, making various payouts after patrons pay $2 and press a start button. The random variable is the amount of the payout, in dollars, in one play of the machine. Each payout can be assumed to be independent of other payouts.

The probability, , that the machine makes a certain payout, , is shown in the table below.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Payout ($) | 0 | 1 | 2 | 5 | 10 | 20 | 50 | 100 |
| Probability | 0.25 | 0.45 | 0.2125 | 0.0625 | 0.0125 | 0.005 | 0.005 | 0.0025 |

(a) Determine the probability that

(i) in one play of the machine, a payout of more than $1 is made. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states probability |

(ii) in ten plays of the machine, it makes a payout of $5 no more than once. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates binomial distribution  ✓ calculates probability |

(iii) in five plays of the machine, the second payout of $1 occurs on the fifth play.

(3 marks)

|  |
| --- |
| **Solution** |
| First payout in one of four plays:  Second payout: |
| **Specific behaviours** |
| ✓ uses first and second event  ✓ calculates for first event  ✓ calculates for both events |

(b) Calculate the mean and standard deviation of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ mean  ✓ sd |

(c) In the long run, what percentage of the player's money is returned to them? (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses mean and payment  ✓ calculates percentage |

Question 6 (10 marks)

Let the random variable be the number of vowels in a random selection of four letters from those in the word LOGARITHM, with no letter to be chosen more than once.

(a) Complete the probability distribution of below. (1 mark)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses sum of probabilities |

(b) Show how the probability for was calculated. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses combinations for numerator  ✓ uses combinations for denominator and simplifies |

(c) Determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ obtains numerator  ✓ obtains denominator and simplifies |

Let event occur when no vowels are chosen in random selection of four letters from those in the word LOGARITHM.

(d) State . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates probability |

(e) Let be a Bernoulli random variable with parameter . Determine the mean and standard deviation of . (2 marks)

|  |
| --- |
| **Solution** |
| is a Bernoulli rv, so |
| **Specific behaviours** |
| ✓ indicates Bernoulli rv and states mean  ✓ states sd |

(f) Determine the probability that occurs no more than twice in ten random selections of four letters from those in the word LOGARITHM. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates binomial distribution with parameters  ✓ calculates probability |

Question 7 (9 marks)

75% of the avocados produced by a farm are known to be first grade, the rest being second grade. Trays of 24 avocados are filled at random in a packing shed and sent to market.

Let the random variable be the number of first grade avocados in a single tray.

(a) Explain why is a discrete random variable, and identify its probability distribution.

(2 marks)

|  |
| --- |
| **Solution** |
| is a DRV as it can only take integer values from 0 to 24.  follows a binomial distribution: |
| **Specific behaviours** |
| ✓ explanation using discrete values  ✓ identifies binomial, with parameters |

(b) Calculate the mean and standard deviation of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ mean, ✓ standard deviation |

(c) Determine the probability that a randomly chosen tray contains

(i) 18 first grade avocados. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ probability |

(ii) more than 15 but less than 20 first grade avocados. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses correct bounds  ✓ probability |

(d) In a random sample of 1000 trays, how many trays are likely to have fewer first grade than second grade avocados. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ identifies upper bound and calculates probability  ✓ calculates whole number of trays |